

12-2 Matrix Multiplication

Objectives: To multiply matrices using scalar and matrix multiplication

Common Core Content Standards:

N.VM.6 Use matrices to represent and manipulate data ...

N.VM.7 Multiply matrices by scalars to produce new matrices ...

Also **N.VM.8** and **N.VM.9**

_____ multiplication is where a real number is distributed across a matrix.



Key Concept Scalar Multiplication

To multiply a matrix by a scalar c , multiply each element of the matrix by c .

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \quad cA = \begin{bmatrix} ca_{11} & ca_{12} & ca_{13} \\ ca_{21} & ca_{22} & ca_{23} \end{bmatrix}$$

Example 1: Using Scalar Products

If $A = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \\ -2 & -4 \end{bmatrix}$, what is $3A - B$?



Properties Scalar Multiplication

If A and B are $m \times n$ matrices, c and d are scalars, and O is the $m \times n$ zero matrix, then

Example

cA is an $m \times n$ matrix

$$(cd)A = c(dA)$$

$$c(A + B) = cA + cB$$

$$(c + d)A = cA + dA$$

$$1 \cdot A = A$$

$$0 \cdot A = O \text{ and } c \cdot O = O$$

Property

Closure Property

Associative Property of Multiplication

Distributive Properties

Multiplicative Identity Property

Multiplicative Properties of Zero

Example 2: Solving a Matrix Equation with Scalars

What is the solution of $-2 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -2 & -14 \\ -2 & 6 \end{bmatrix}$?

_____ multiplication is where you multiply the elements of a row from the first matrix by the corresponding elements of a column from the second matrix. Then add the products.



Key Concept Matrix Multiplication

To find element c_{ij} of the product matrix AB , multiply each element in the i th row of A by the corresponding element in the j th column of B . Then add the products.

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Example 3: Multiplying Matrices

If $A = \begin{bmatrix} -4 & 0 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$,

what is a.) AB and b.) BA ?

Example 4: Applying Matrix Multiplication

A library has three printers. The cost of printing from printer A is 3 cents per page, from printer B is 6 cents per page, and from printer C is 14 cents per page. During October and November, the librarian recorded the number of pages printed on each printer, as shown in the table. Using Matrix multiplication, what was the monthly cost of operating the printers for October and November?

	Oct	Nov
Printer A	584	598
Printer B	549	610
Printer C	159	185

You can multiply two matrices A and B only if the number of columns of A is equal to the number of rows of B.

Example 5: Determining Whether Product Matrices Exist

Does either product AB or BA exist?

$$A = \begin{bmatrix} -5 & 0 \\ 3 & -2 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 & -1 \\ 4 & 2 & 5 \end{bmatrix}$$