

12-3

Practice

Form K

Determinants and Inverses

Determine whether the following matrices are multiplicative inverses.

1. $\begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix}$, $\begin{bmatrix} 2 & -0.5 \\ -3 & 1 \end{bmatrix}$ 2. $\begin{bmatrix} 2 & -4 & 1 \\ 6 & -3 & -7 \\ 9 & 5 & -2 \end{bmatrix}$, $\begin{bmatrix} 8 & 3 & -3 \\ -9 & 2 & 7 \\ 4 & -1 & -6 \end{bmatrix}$ 3. $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$, $\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

Evaluate the determinant of each matrix.

To start, write the formula for the determinant of a 2×2 matrix.

4. $\begin{bmatrix} 4 & 1 \\ -5 & 3 \end{bmatrix}$

5. $\begin{bmatrix} -1 & 3 & -4 \\ 6 & -2 & 8 \\ 5 & 7 & -3 \end{bmatrix}$

6. $\begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix}$

7. $\begin{bmatrix} 2 & 0 & -4 \\ -1 & 3 & 2 \\ -2 & 1 & 4 \end{bmatrix}$

8. $\begin{bmatrix} -2 & -3 \\ 5 & 7 \end{bmatrix}$

9. $\begin{bmatrix} 3 & 4 & -1 \\ 1 & 8 & 3 \\ 5 & 2 & 2 \end{bmatrix}$

10. **Error Analysis** Your friend evaluated the determinant of the matrix $\begin{bmatrix} -6 & -7 \\ 3 & 2 \end{bmatrix}$ and got -9 . What error did your friend make, and what is the correct determinant?

11. **Open-Ended** Write a 2×2 matrix with a determinant of zero.

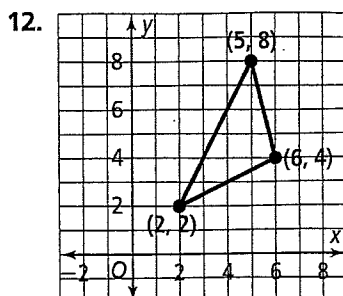
12-3

Practice (continued)

Form K

Determinants and Inverses

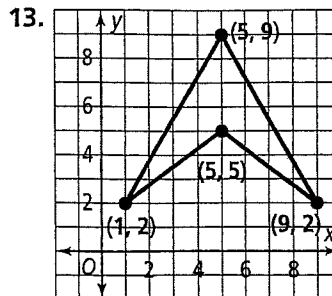
Use matrices to find the areas of the following figures. Express your answers in square units.



$$\text{Area} = \frac{1}{2} |\det A|$$

$$\text{Area} = \frac{1}{2} \det \begin{bmatrix} 2 & 2 & 1 \\ 6 & 4 & 1 \\ 5 & 8 & 1 \end{bmatrix}$$

$$\text{Area} = \boxed{}$$



Find the inverse of each matrix, if one exists.

To start, find the determinant of the matrix.

14. $A = \begin{bmatrix} 6 & 2 \\ 2 & 1 \end{bmatrix}$

15. $A = \begin{bmatrix} 5 & 8 \\ 3 & 5 \end{bmatrix}$

16. $A = \begin{bmatrix} 10 & 5 \\ 4 & 2 \end{bmatrix}$

$$\det A = 6(1) - 2(2) = 2$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -2 \\ -2 & 6 \end{bmatrix}$$

17. Your aunt's checking account number is 6143-0571-2943-3072. Use the coding

matrix $C = \begin{bmatrix} -2 & 1 \\ -1 & 3 \end{bmatrix}$ to encode the account number.

12-4

Practice

Form K

Inverse Matrices and Systems

Solve each matrix equation.

To start, find the determinant of the coefficient matrix.

1. $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} X = \begin{bmatrix} 11 & 24 \\ 5 & 10 \end{bmatrix}$

2. $\begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$

$$\det \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = 5(1) - 2(2) = 1$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 11 & 24 \\ 5 & 10 \end{bmatrix} = \boxed{}$$

Write each system as a matrix equation. Identify the coefficient matrix, the variable matrix, and the constant matrix.

3. $\begin{cases} 3x + y = 9 \\ 2x - 4y = -8 \end{cases}$

4. $\begin{cases} a + 4b = 13 \\ 3a + 2b = 19 \end{cases}$

5. $\begin{cases} 5x + 3y = 35 \\ 2x = 38 - 6y \end{cases}$

6. $\begin{cases} 3x + y = 9 \\ x - y + 4z = -5 \\ 3y + 2z = 7 \end{cases}$

7. $\begin{cases} 4x = 2 - 2y \\ 3y = -12 - x \end{cases}$

8. $\begin{cases} 2a - 2c = -6 - b \\ 4a = 10 + c \\ 3c = 8 - 5b \end{cases}$

12-4 Practice (continued)

Inverse Matrices and Systems

Form K

Solve each system of two equations using a matrix equation.

$$9. \begin{cases} 3x - y = 16 \\ 5x - 9y = 12 \end{cases}$$

$$\begin{bmatrix} 3 & -1 \\ 5 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{22} \begin{bmatrix} -9 & 1 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{9}{22} & -\frac{1}{22} \\ \frac{5}{22} & -\frac{3}{22} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{9}{22} & -\frac{1}{22} \\ \frac{5}{22} & -\frac{3}{22} \end{bmatrix} \begin{bmatrix} 16 \\ 12 \end{bmatrix} = \boxed{}$$

$$10. \begin{cases} y = 32 - 4x \\ -2x = -2 - 3y \end{cases}$$

$$11. \begin{cases} 3a + 4b = -3 \\ 2a + 3b = -1 \end{cases}$$

$$12. \begin{cases} 2b = 3a - 14 \\ 2b = -20 + 4a \end{cases}$$

Solve each system of three equations using a matrix equation.

$$13. \begin{cases} 3x + 5y = 19 \\ x + 3y - 4z = 1 \\ 6x + 8z = 12 \end{cases}$$

$$14. \begin{cases} 3b + 5c = 7 \\ 9 = a - 2c \\ 4a + 7c = 17 + b \end{cases}$$

$$15. \begin{cases} 6y = 8 - x \\ x + y + z = -8 \\ 7y - 3z = 32 \end{cases}$$