Algebra 2A Notes

Name: ______________________________

12-3 Determinants and Inverses

Date: ____________ Hr: ____________

Objectives: To find the inverse of a matrix

Common Core Content Standards:

N.VM.10 ... The determinant of a square matrix is nonzero if and only if the matrix has a multiplication inverse.

N.VM.12 Work with 2 x 2 matrices as a transformation of the plane...

The product of a matrix and its multiplicative ________________________ matrix is the multiplicative __________________________ matrix. Not all matrices have inverse matrices.

A ___________________________ matrix is a matrix with the same number of rows and columns. While there is a multiplicative identity matrix for any square matrix, not all square matrices have multiplicative inverses.

take note

Key Concepts Identity and Multiplicative Inverse Matrices

For an $n \times n$ matrix, the **multiplicative identity matrix** is an $n \times n$ matrix I, or I_n, with 1's along the main diagonal and 0's elsewhere.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and so forth.}$$

If A and B are square matrices and $AB = BA = I$, then B is the **multiplicative inverse matrix** of A, written A^{-1}.

Example 1: Determining Whether Matrices are Inverses

$$If\ A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & -1 \\ 2 & 1 & 1 \end{bmatrix} and\ B = \begin{bmatrix} \frac{4}{3} & \frac{1}{3} & -1 \\ -1 & 0 & 1 \\ -\frac{5}{3} & -\frac{2}{3} & 2 \end{bmatrix},\ are\ A\ an\ B\ inverses?$$

take note

Key Concept **Determinants of 2 × 2 and 3 × 3 Matrices**

The **determinant** of a 2 × 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is det $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$.

The determinant of a 3 × 3 matrix $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is

$$a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - (a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1)$$

Visualize the pattern this way: $\left[\begin{array}{ccc|cc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array}\right]$ a copy of the first two columns

Example 2: Evaluating the Determinants of Matrices

What are the following determinants?

a.) det $\begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}$

b.) det $\begin{bmatrix} 3 & 5 & -1 \\ 1 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

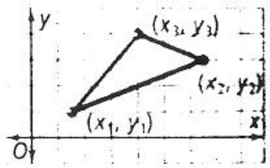
You can use determinants to find the areas of polygons. Since all polygons can be divided into 'angles, all you need is a way to find the area of a triangle.

take note

Key Concept Area of a Triangle

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\text{Area} = \frac{1}{2}|\det A| \text{ where } A = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$



Example 3: **Finding the Area of a Polygon**

As part of a remodeling project, you want to paint a triangular area on a cement floor that is marked along the wall with decorative stones using every meter. Using the stones as a reference, you determine the coordinates of the vertices of the area you want to paint are (4, 6), (12, 9), and (7, 11). What is the area of the triangle?

The determinant of a matrix can help you determine whether the matrix has an inverse and, if it exists, to find the inverse.

***Example 4:* Finding the Inverse of a Matrix**

Does the matrix $A = \begin{bmatrix} 4 & -4 \\ -3 & 6 \end{bmatrix}$ have an inverse? If it does, what is A^{-1}?

take note

Key Concept **Inverse of a 2 × 2 Matrix**

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

If $\det A = 0$, then A is a **singular matrix** and has no inverse.

If $\det A \neq 0$, then the inverse of A, written A^{-1}, is

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

***Example 5:* Encoding and Decoding with Matrices**

You stored your credit card numbers in a file after they were coded by the matrix $\begin{bmatrix} -5 & 3 \\ 3 & -7 \end{bmatrix}$. One of the numbers in the -1, -14, -16, -22, 10, 10, -31, -29, -15, -2, -6, 8, -32, -32, 3, 7. What is the original credit card number?