

## 5-1 Polynomial Functions

## Objectives:

- To classify polynomials.
- To graph polynomial functions and describe end behavior.

## Common Core Content Standard:

**F.IF.7.c** Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.

## Also A.SSE.1.a

A polynomial function has distinguishing “behaviors.” You can look at its algebraic form and know something about its graph. You can look at its graph and know something about its algebraic form.

A \_\_\_\_\_ is a real number, a variable, or a product of a real number and one or more variables with whole number exponents. The degree of a monomial in one variable is the exponent of the variable.

A \_\_\_\_\_ is a monomial or a sum of monomials. The degree of a polynomial in one variable is the greatest degree among its monomial terms.

**Key Concept** Standard Form of a Polynomial Function

The **standard form of a polynomial function** arranges the terms by degree in descending numerical order.

A polynomial function  $P(x)$  in standard form is

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where  $n$  is a nonnegative integer and  $a_n, \dots, a_0$  are real numbers.

$$P(x) = 1x^3 + 3x^2 + 5x + 2$$

Cubic term    Quadratic term    Linear term    Constant term

You can classify a polynomial by its degree or by its number of terms. Polynomials of degrees zero through five have specific names, as shown in this table.

0	constant	5	1	monomial
1	linear	$x + 4$	2	binomial
2	quadratic	$4x^2$	1	monomial
3	cubic	$4x^3 - 2x^2 + x$	3	trinomial
4	quartic	$2x^4 + 5x^2$	2	binomial
5	quintic	$-x^5 + 4x^2 + 2x + 1$	4	polynomial of 4 terms

### Example 1: Classifying Polynomials

Write  $-3x + 4x^2 + 7x - 3$  in standard form. What is the classification of this polynomial by its degree? by its number of terms?

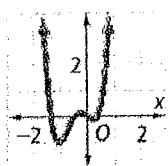
The degree of a polynomial function affects the shape of its graph and determines the maximum number of \_\_\_\_\_, or places where the graph changes direction. It also affects the \_\_\_\_\_, or the directions of the graph to the far left and to the far right.

A function is \_\_\_\_\_ when the y-values increase as x-values increase. A function is \_\_\_\_\_ when the y-values decrease as x-values decrease.



#### Key Concept Polynomial Functions

$$y = 4x^4 + 6x^3 - x$$

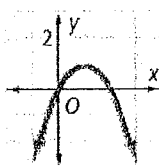


End Behavior: Up and Up

Turning Points:  $(-1.07, -1.04)$ ,  $(-0.27, 0.17)$ , and  $(0.22, -0.15)$

The function is decreasing when  $x < -1.07$  and  $-0.27 < x < 0.22$ . The function increases when  $-1.07 < x < -0.27$  and  $x > 0.22$ .

$$y = -x^2 + 2x$$

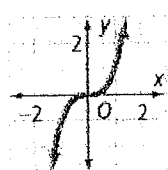


End Behavior: Down and Down

Turning Point:  $(1, 1)$

The function is increasing when  $x < 1$  and is decreasing when  $x > 1$ .

$$y = x^3$$

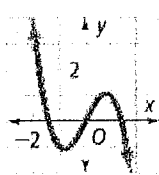


End Behavior: Down and Up

Zero turning points.

The function is increasing for all x.

$$y = -x^3 + 2x$$



End Behavior: Up and Down

Turning Points:  $(-0.82, -1.09)$  and  $(0.82, 1.09)$

The function is decreasing when  $x < -0.82$  and when  $x > 0.82$ . The function is increasing when  $-0.82 < x < 0.82$ .

In general, the graph of a polynomial function of degree  $n$  ( $n \geq 1$ ) has at most \_\_\_\_\_ turning points. The graph of a polynomial function of odd degree has an \_\_\_\_\_ number of turning points. The graph of a polynomial function of even degree has an \_\_\_\_\_ number of turning points.

### Example 2: Describing End Behavior of Polynomial Functions

Consider the leading term of  $y = 3x^4 - 2x^3 + x - 1$ . What is the end behavior of the graph?

### Example 3: Graphing Cubic Functions

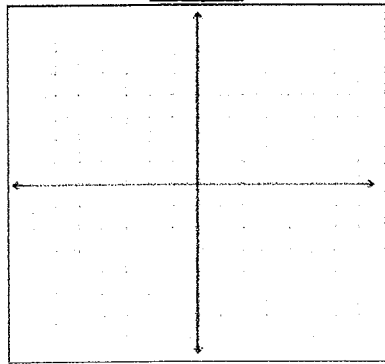
What is the graph of each cubic function? Describe the graph, including end behavior, turning points, and increasing/decreasing intervals.

a.)  $y = \frac{1}{2}x^3$

b.)  $y = 3x - x^3$

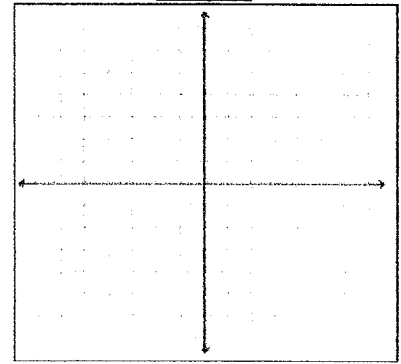
Step 1

Step 2



Step 1

Step 2



Step 3

Step 3

If the first set of polynomial function outputs are constant, the function is \_\_\_\_\_. If the second differences (but not the first) are constant, the function is \_\_\_\_\_. If the third differences (but not the second) are constant, the function is \_\_\_\_\_, and so on.

**Example 4: Using Differences to Determine Degree**

What is the degree of the polynomial function that generates the data shown in the table?

$x$	$y$
-2	-13
-1	-4
0	-1
1	2
2	11
3	32
4	71