Name _____ Class _____ Date _____

# 5-1 Reteaching

## Polynomial Functions

**Problem**

What is the classification of the following polynomial by its degree? by its number of terms? What is its end behavior? $5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

**Step 1** Write the polynomial in standard form. First, combine any like terms. Then, place the terms of the polynomial in descending order from greatest exponent value to least exponent value.

$5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

$8x^4 - 3x + 3x^6 + 9x^3 - 12$ Combine like terms.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ Place terms in descending order.

**Step 2** The degree of the polynomial is equal to the value of the greatest exponent. This will be the exponent of the first term when the polynomial is written in standard form.

$(3x^6) + 8x^4 + 9x^3 - 3x - 12$ The first term is $3x^6$.

$3x^{(6)}$ The exponent of the first term is 6.

This is a sixth-degree polynomial.

**Step 3** Count the number of terms in the simplified polynomial. It has 5 terms.

**Step 4** To determine the end behavior of the polynomial (the directions of the graph to the far left and to the far right), look at the degree of the polynomial ($n$) and the coefficient of the leading term ($a$).

If $a$ is positive and $n$ is even, the end behavior is up and up.

If $a$ is positive and $n$ is odd, the end behavior is down and up.

If $a$ is negative and $n$ is even, the end behavior is down and down.

If $a$ is negative and $n$ is odd, the end behavior is up and down.

The leading term in this polynomial is $3x^6$.
$a$ (+3) is positive and $n$ (6) is even, so the end behavior is up and up.

## Exercises

**What is the classification of each polynomial by its degree? by its number of terms? What is its end behavior?**

1. $8 - 6x^3 + 3x + x^3 - 2$
2. $15x^7 - 7$
3. $2x - 6x^2 - 9$

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# 5-1 Reteaching (continued)

## Polynomial Functions

### Problem

What is the degree of the polynomial function that generates the data shown at the right? What are the differences when they are constant?

| $x$ | $y$ |
|---|---|
| −3 | 52 ($y_1$) |
| −2 | 18 ($y_2$) |
| −1 | 2 ($y_3$) |
| 0 | −2 ($y_4$) |
| 1 | 0 ($y_5$) |
| 2 | 2 ($y_6$) |
| 3 | −2 ($y_7$) |

To find the degree of a polynomial function from a data table, you can use the differences of the $y$-values.

**Step 1** Determine the values of $y_2 - y_1, y_3 - y_2, y_4 - y_3, y_5 - y_4, y_6 - y_5, y_7 - y_6$. These are called the first differences. Make a new column using these values.

**Step 2** Continue determining differences until the $y$-values are all equal. The quantity of differences is the degree of the polynomial function.

The third differences are all equal so this is a third degree polynomial function. The value of the third differences is −6.

| $x$ | $y$ | 1st diff. |
|---|---|---|
| −3 | 52 ($y_1$) | |
| | | −34 ($y_8$) |
| −2 | 18 ($y_2$) | |
| | | −16 ($y_9$) |
| −1 | 2 ($y_3$) | |
| | | −4 ($y_{10}$) |
| 0 | −2 ($y_4$) | |
| | | 2 ($y_{11}$) |
| 1 | 0 ($y_5$) | |
| | | 2 ($y_{12}$) |
| 2 | 2 ($y_6$) | |
| | | −4 ($y_{13}$) |
| 3 | −2 ($y_7$) | |

| $x$ | $y$ | 1st diff. | 2nd diff. | 3rd diff. |
|---|---|---|---|---|
| −3 | 52 ($y_1$) | | | |
| | | −34 ($y_8$) | | |
| −2 | 18 ($y_2$) | | 18 | |
| | | −16 ($y_9$) | | −6 |
| −1 | 2 ($y_3$) | | 12 | |
| | | −4 ($y_{10}$) | | −6 |
| 0 | −2 ($y_4$) | | 6 | |
| | | 2 ($y_{11}$) | | −6 |
| 1 | 0 ($y_5$) | | 0 | |
| | | 2 ($y_{12}$) | | −6 |
| 2 | 2 ($y_6$) | | −6 | |
| | | −4 ($y_{13}$) | | |
| 3 | −2 ($y_7$) | | | |

## Exercises

**What is the degree of the polynomial function that generates the data in the table? What are the differences when they are constant?**

4.

| $x$ | $y$ |
|---|---|
| −3 | 216 |
| −2 | 24 |
| −1 | 0 |
| 0 | 0 |
| 1 | 0 |
| 2 | −24 |
| 3 | −216 |

5.

| $x$ | $y$ |
|---|---|
| −3 | −101 |
| −2 | −37 |
| −1 | −11 |
| 0 | −5 |
| 1 | −1 |
| 2 | 19 |
| 3 | 73 |

6.

| $x$ | $y$ |
|---|---|
| −3 | 6 |
| −2 | 26 |
| −1 | 8 |
| 0 | 0 |
| 1 | 2 |
| 2 | −34 |
| 3 | −204 |