

5-² Polynomials, Linear Factors, and Zeros

Objectives:

- To analyze the factored form of a polynomial.
- To write a polynomial function from its zeros.

Common Core Content Standard:

F.IF.7.c Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.

Also A.APR.3

If $P(x)$ is a polynomial function, the solutions of the related polynomial equation $P(x) = 0$ are the _____ of the function. Finding the zeros of a polynomial function will help you factor the polynomial, graph the function, and solve the related polynomial equation.

Example 1: Writing a Polynomial in Factored Form

What is the factored form of $x^3 + x^2 - 12x$?



Key Concepts Roots, Zeros, and x-intercepts

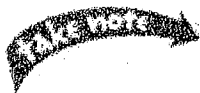
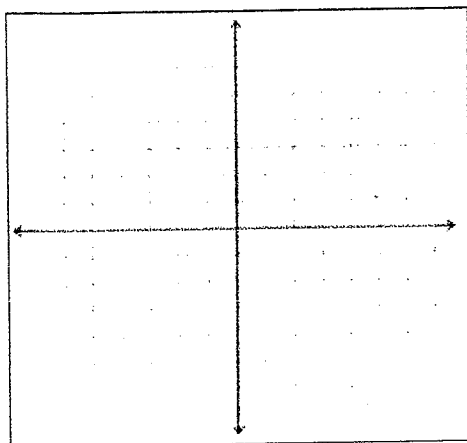
The following are equivalent statements about a real number b and a polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

- $x - b$ is a linear factor of the polynomial $P(x)$.
- b is a zero of the polynomial function $y = P(x)$.
- b is a root (or solution) of the polynomial equation $P(x) = 0$.
- b is an x-intercept of the graph of $y = P(x)$.

Example 2: Finding Zeros of a Polynomial Function

What are the zeros of $y = (x - 3)(x + 4)(x - 1)$? Graph the function.



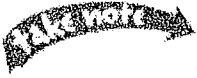
Theorem Factor Theorem

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Example 3: Writing a Polynomial Function from its Zeros

What is a cubic polynomial function in standard form with zeros 1, -1, and 4?

The _____ of a zero of a polynomial function is the number of times the related linear factor is repeated in the factored form of the polynomial.



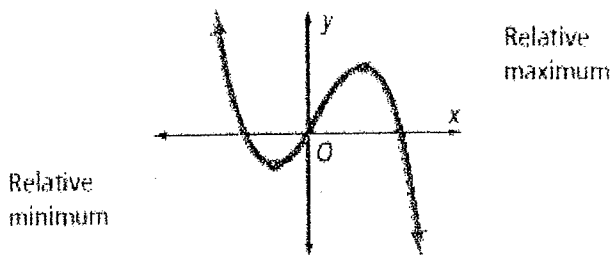
Key Concept How Multiple Zeros Affect a Graph

If a is a zero of multiplicity n in the polynomial function $y = P(x)$, then the behavior of the graph at the x -intercept a will be close to linear if $n = 1$, close to quadratic if $n = 2$, close to cubic if $n = 3$, and so on.

Example 4: Finding the Multiplicity of a Zero

What are the zeros of $f(x) = x^3 - 5x^2 + 3x + 9$? What are the multiplicities? How does the graph behave at these zeros?

A _____ is the value of the function at an up-to-down turning point. A _____ is the value of the function at a down-to-up turning point.



Example 5: Identifying a Relative Maximum and Minimum

What are the relative maximum and minimum of $f(x) = x^3 - 9x$? Round to the nearest tenth.

Example 6: Using a Polynomial Function to Maximize Volume

A designer wants to make a rectangular prism box with maximum volume, while keeping the sum of its length, width, and height 12 in. The length must be 3 times the height. What should each dimension be?

