

## 6-4 Rational Exponents

**Objectives:**

- To simplify expressions with rational exponents

**Common Core Standards**

**Reviews N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents. Also reviews **N.RN.1**.

You can write a radical expression in an equivalent form using a fractional (rational) exponent instead of a radical sign.

**Example 1: Simplifying Expressions with Rational Exponents**

What is the simplified form of each expression?

a.)  $625^{1/4}$

b)  $12^{1/2} \cdot 12^{1/2}$

c.)  $12^{1/3} \cdot 18^{1/3}$

**Take note****Key Concept Rational Exponent**

If the  $n$ th root of  $a$  is a real number,  $m$  is an integer, and  $\frac{m}{n}$  is in lowest terms, then

$$a^{1/n} = \sqrt[n]{a} \text{ and } a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m. \quad \text{If } m \text{ is negative, } a \neq 0.$$

## Example 2: Converting Between Exponential and Radical Forms

a.) What are  $-2y^{4/5}$  and  $d^{-1.5}$  in radical form?

b.) What are  $\sqrt{ab^3}$  and  $\sqrt[3]{w^2}$  in exponential form?

## Example 3: Using Rational Exponents

For a  $\frac{1}{4}$  in. thick cookie, the diameter  $d$  of the cookie is related to the diameter  $c$  of the ball of dough by  $d = 1.6c^{3/2}$ . What is the diameter to the nearest tenth of an inch of a cookie made from a 2 in. ball of dough?

All the properties of integer exponents apply to rational exponents.

**Take note**

### Properties Properties of Rational Exponents

Let  $m$  and  $n$  represent rational numbers. Assume that no denominator equals 0.

Property	Example	Property	Example
$a^m \cdot a^n = a^{m+n}$	$8^{\frac{1}{3}} \cdot 8^{\frac{2}{3}} = 8^{\frac{1}{3} + \frac{2}{3}} = 8^1 = 8$	$a^{-m} = \frac{1}{a^m}$	$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$
$(a^m)^n = a^{mn}$	$(5^{\frac{1}{2}})^4 = 5^{\frac{1}{2} \cdot 4} = 5^2 = 25$	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^{\frac{3}{2}}}{7^{\frac{1}{2}}} = 7^{\frac{3}{2} - \frac{1}{2}} = 7^1 = 7$
$(ab)^m = a^m b^m$	$(4 \cdot 5)^{\frac{1}{2}} = 4^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 2 \cdot 5^{\frac{1}{2}}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{27}\right)^{\frac{1}{3}} = \frac{5^{\frac{1}{3}}}{27^{\frac{1}{3}}} = \frac{5^{\frac{1}{3}}}{3}$

Recall from Lesson 6-2 that you simplified products or quotients involving radical expressions only when they had the same index. However, you can combine radical expressions with different indexes if you convert them to expressions with rational exponents.

**Example 4: Combining Radical Expressions**

What is  $\sqrt[6]{10}(\sqrt[3]{10})$  in simplest form?

---

You can simplify a number with a rational exponent by using the properties of exponents or by converting the expression to a radical expression.

**Example 5: Simplifying Numbers with Rational Exponents**

What is each number in simplest form?

a.)  $(-64)^{5/3}$

b.)  $32^{-1.2}$

To write an expression with rational exponents in simplest form, write every exponent as a positive number.

**Example 6: Writing Expressions in Simplest Form**

What is each expression in simplest form?

a.)  $(9y\sqrt{x})^{3/2}$

b.)  $(27x^6y^9)^{-1/3}$

