

7-3 Logarithmic Functions as Inverses

Objectives:

- To write and evaluate logarithmic expressions
- To graph logarithmic functions

Common Core Content Standard:

F.BF.4.a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write ... the inverse.

F.IF.7.e Graph exponential ... functions, showing intercepts and end behavior ...

Also **A.SSE.1.b**, **F.IF.8**, **F.IF.9**

In this lesson, you will find ways to express all numbers as powers of a common base.

Take note

Key Concept Logarithm

A logarithm base b of a positive number x satisfies the following definition.

For $b > 0$, $b \neq 1$, $\log_b x = y$ if and only if $b^y = x$.

You can read $\log_b x$ as "log base b of x ." In other words, the logarithm y is the exponent to which b must be raised to get x .

The exponent y in the expression b^y is the logarithm in the equation $\log_b x = y$. The base b in b^y and the base b in $\log_b x$ are the same. In both, $b \neq 1$ and $b > 0$.

Since $b \neq 1$, and $b > 0$, it follows that $b^y > 0$. Since $b^y = x$, then $x > 0$, so $\log_b x$ is defined only for $x > 0$.

Because $y = b^x$ and $y = \log_b x$ are inverse functions, their compositions map a number a to itself. In other words, $b^{\log_b a} = a$ for $a > 0$ and $\log_b b^a = a$ for all a .

You can use the definition of a logarithm to write exponential equations in logarithmic form.

Example 1: Writing Exponential Equations in Logarithmic Form.

What is the logarithmic form of each equation?

a.) $8^0 = 1$

b.) $4^3 = 64$

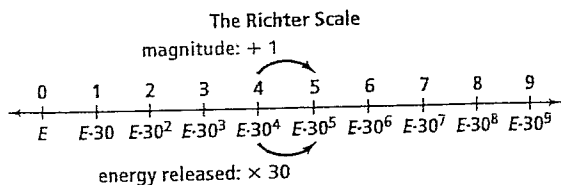
You can use the exponential form to help you evaluate logarithms.

Example 2: Evaluating a Logarithm

What is the value of $\log_{16}64$?

A _____ logarithm is a logarithm with base 10. You can rewrite a common logarithm $\log_{10}x$ simply as $\log x$, without showing the 10

Many measurements of physical phenomena have such a wide range of values that the reported measurements are logarithms (exponents) of the values, not the values themselves. When you use the logarithm of a quantity instead of the quantity, you are using a _____ scale. It gives logarithmic measurements of the earthquake magnitude.

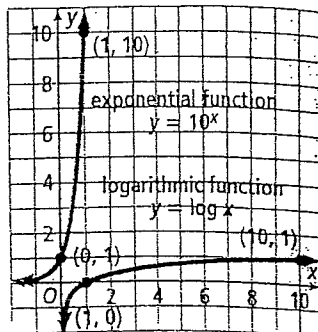


Example 3: Using a Logarithmic Scale

In 1995, an earthquake in Mexico registered 8.0 on the Richter scale. In 2001, an earthquake of magnitude 6.8 shook Washington state. How many times more intense was the 1995 earthquake than the 2001 earthquake? Use the formula $\log \frac{I_1}{I_2} = M_1 - M_2$ which compares the intensity levels of earthquakes where I is the intensity level determined by a seismograph, and M is the magnitude on a Richter scale.

A logarithmic function is the inverse of an exponential function. The graph shows $y = 10^x$ and its inverse $y = \log x$. Note that $(0, 1)$ and $(1, 10)$ are on the graph of $y = 10^x$, and that $(1, 0)$ and $(10, 1)$ are on the graph of $y = \log x$.

Recall that the graphs of inverse functions are reflections of each other across the line $y = x$. You can graph $y = \log_b x$ as the inverse of $y = b^x$.



Example 4: Graphing a Logarithmic Function

What is the graph of $y = \log_5 x$? Describe the domain and range and identify the y-intercept and the asymptote.

Take note

Concept Summary Families of Logarithmic Functions

Parent functions: $y = \log_b x, b > 0, b \neq 1$

Stretch ($|a| > 1$)
Compression (Shrink) ($0 < |a| < 1$)
Reflection ($a < 0$) in x-axis

$$y = a \log_b x$$

Translations (horizontal by h ; vertical by k) $y = \log_b (x - h) + k$

All transformations together $y = a \log_b (x - h) + k$

Example 5: Transforming $y = \log_b x$

How does the graph of $y = \frac{3}{4} \log_x - 2$ compare to the graph of the parent function?

