# lgebra 2C Notes

Name: ______________________________

## -3 Rational Functions and Their Graphs

Date: __________ Hr: __________

**ectives:**

- To identify properties of rational functions.
- To graph rational functions.

**ommon Core Standards**

**.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros o construct a rough graph of the function ...

**.BF.1.b** Combine standard function types using arithmetic operations.

\ ______________________ function is a ratio of two polynomial functions.

Rational functions have gaps at each zero of the denominator. The gap could be a one-point _______________ in the graph, or it could be the location of a vertical _______________ for the graph.

A graph is ____________________ if it has no jumps, breaks, or holes. You can draw the graph and your pencil never leaves the paper.

A onal function with holes and/or vertical asymptotes is discontinuous. A hole is called a _______________ discontinuity and a vertical asymptote is known as a _____________________ discontinuity.

take note

**Key Concept** Point of Discontinuity

If $a$ is a real number for which the denominator of a rational function $f(x)$ is zero, then $a$ is not in the domain of $f(x)$. The graph of $f(x)$ is not continuous at $x = a$ and the function has a **point of discontinuity** at $x = a$.

The graph of $y = \frac{(x+3)(x+2)}{x+2}$ has a **removable discontinuity** at $x = -2$. The hole in the graph is called a removable discontinuity because you could make the function continuous by redefining it at $x = -2$ so that $f(-2) = 1$.

The graph of $y = \frac{x+4}{x-2}$ has a **non-removable discontinuity** at $x = 2$. There is no way to redefine the function at 2 to make the function continuous.

/hen looking for discontinuities, it is first necessary to factor the numerator and denominator. A hole ill then exist at a common factor between the numerator and denominator. A vertical asymptote will xist at a non-common factor of the denominator.

## *xample 1:* Finding Points of Discontinuity

Vhat are the domain and points of discontinuity of $\frac{x^2+4x+4}{x+2}$? Are the points of discontinuity removable r non-removable? What are the x- and y- intercepts?

ake note

**Key Concept** Vertical Asymptotes of Rational Functions

The graph of the rational function $f(x) = \frac{P(x)}{Q(x)}$ has a vertical asymptote at each real zero of $Q(x)$ if $P(x)$ and $Q(x)$ have no common zeros. If $P(x)$ and $Q(x)$ have $(x - a)^m$ and $(x - a)^n$ as factors, respectively and $m < n$, then $f(x)$ also has a vertical asymptote at $x = a$.

## *xample 2:* Finding Vertical Asymptotes

Vhat are the vertical asymptotes for the graph of $y = \frac{x-3}{(x^2-3x+2)(x^2-7x+12)}$?

While the graph of a rational function can have any number of vertical asymptotes, it can have no more than one horizontal asymptote.

take note

**Key Concept** Horizontal Asymptote of a Rational Function

To find the horizontal asymptote of the graph of a rational function, compare the degree of the numerator $m$ to the degree of the denominator $n$.

If $m < n$, the graph has horizontal asymptote $y = 0$ (the $x$-axis).

If $m > n$, the graph has no horizontal asymptote.

If $m = n$, the graph has horizontal asymptote $y = \frac{a}{b}$ where $a$ is the coefficient of the term of greatest degree in the numerator and $b$ is the coefficient of the term of greatest degree in the denominator.

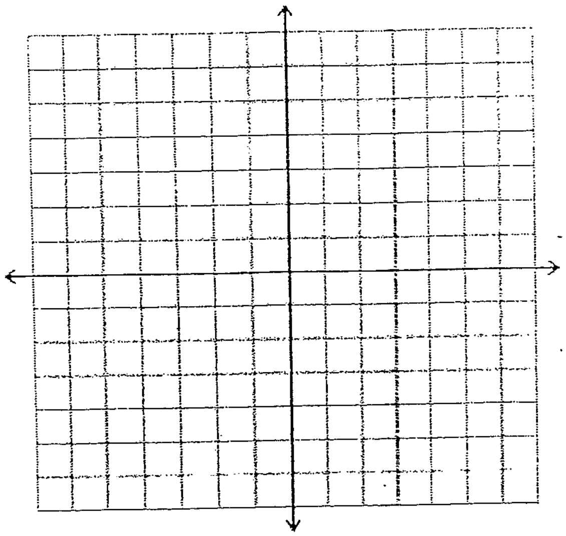
*Example 3:* **Finding Horizontal Asymptotes**

What is the horizontal asymptote for the graph of $y = \frac{x^2+1}{-3x+6}$?

You can get a reasonable graph for a rational function by finding all intercepts and asymptotes. Sometimes you will also have to plot a few extra points to get a good sense of the shape of the graph.

## *Example 4:* Graphing a Rational Function

What is the graph of the rational function $y = \frac{x^2+1}{-3x+6}$?



## *Example 5:* Using a Rational Function

Whole milk contains 3.7% fat. You want to add 2% fat milk to 5 fl. oz. of whole milk to make 3%-fat milk. The function $y = \frac{(5)(0.037)+x(0.02)}{5+x}$ gives the percentage of fat in a new concentration after you add x fluid ounces of the 2% milk. How many fluid ounces of 2% milk must you add?