

Algebra 2B Notes

Name: _____

9-4 Arithmetic Series

Date: _____ Hr: _____

Objective:

- To define arithmetic series and find their sums

Common Core Content Standard:

Extends F.1F.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

A _____ is the indicated sum of the terms of a sequence.

A _____ series, like a finite sequence, has a first term and a last term, while an _____ series continues without end.

An _____ series is a series whose terms form an arithmetic sequence. When a series has a finite number of terms, you can use a formula involving the first and last term to evaluate the sum.

Take note

Property Sum of a Finite Arithmetic Series

The sum S_n of a finite arithmetic series $a_1 + a_2 + a_3 + \dots + a_n$ is

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where a_1 is the first term, a_n is the n th term, and n is the number of terms.

Example 1: Finding the Sum of a Finite Arithmetic Series

What is the sum of the finite arithmetic series $14 + 17 + 20 + 23 + \dots + 16$?

Example 2: Using the Sum of a Finite Arithmetic Series

There are 30 rows of seats in a large arena. The first row contains 10 seats. Each successive row increases by 3 seats. How many seats are in the last row? How many seats are there in all?

You can use the Greek capital letter sigma, Σ , to indicate a sum. With it, you use *limits* to indicate how many terms you are adding. **Limits** are the least and greatest values of n in the series. You write the limits below and above the Σ to indicate the first and last terms of the series.

For example, you can write the series $3^2 + 4^2 + 5^2 + \dots + 108^2$ as $\sum_{n=3}^{108} n^2$.

Upper limit: the series ends with $n = 108$.

$$\sum_{n=3}^{108} n^2$$

The explicit formula for each term is n^2 .

Lower limit: the series begins with $n = 3$.

For an infinite series, summation notation shows ∞ as the upper limit.

To find the number of terms in a series written in Σ form, subtract the lower limit from the upper limit and add 1.

The number of terms in the series above is $108 - 3 + 1 = 106$.

Example 3: Writing a Series in Summation Notation

What is the summation notation for the series?

a.) $-19 + -14 + -9 + \dots + 221 + 226$

b.) $20 + 18 + 16 + \dots + -24 + -26$

Take note

Key Concept Summation Notation and Linear Functions

If the explicit formula for the n th term in summation notation is a *linear* function of n , then the series is arithmetic. The slope of the linear function is the common difference between terms of the series.

Example 4: Finding the Sum of a Series

What is the sum of the series written in summation notation?

$$\text{a.) } \sum_{n=1}^{27} (-2n + 1)$$

$$\text{b.) } \sum_{n=1}^5 (n^2 + 5)$$

$$\text{c.) } \sum_{n=0}^5 (10^n)$$

Example 5: Using a Graphing Calculator to Find the Sum of a Series

What is the sum of the series written in summation notation?

$$\sum_{n=1}^{85} (n^2 + 4n + 3)$$

