

Algebra 2B Notes

Name: _____

9-5 Geometric Series

Date: _____ Hr: _____

Objective:

- a) To define Geometric series and find their sums

Common Core Content Standard:

A.SSE.4. Derive the formula for the sum of a geometric series (when the common ratio is not 1), and use the formula to solve problems.

A _____ is the sum of the terms of a geometric sequence.

Take note

Key Concept Sum of a Finite Geometric Series

The sum S_n of a finite geometric series $a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1}$, $r \neq 1$, is

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

where a_1 is the first term, r is the common ratio, and n is the number of terms.

Example 1: Finding the Sum of a Finite Geometric Series

What is the sum of the finite geometric series?

a) $4 + 12 + 36 + 108 + 324 + 972 + 2916$

b) $\sum_{n=0}^{11} 3(-1.5)^n$

Example 2: Using the Geometric Series Formula

A game show is offering a prize of 1¢ on the first day, 3¢ on the second day, 9¢ on the third day, etc. What is the total amount of money earned from this prize in two weeks?

The terms of a geometric series grow rapidly when the common ratio is greater than 1. Likewise, they diminish rapidly when the common ratio is between 0 and 1. In fact, they diminish so rapidly that an infinite geometric series has a finite sum.

Take note

Key Concept Infinite Geometric Series

An infinite geometric series with first term a_1 and common ratio $|r| < 1$ has a finite sum

$$S = \frac{a_1}{1 - r}.$$

An infinite geometric series with $|r| \geq 1$ does not have a finite sum.

To say that an infinite series $a_1 + a_2 + a_3 + \dots$, has a sum means that the sequence of partial sums $S_1 = a_1$, $S_2 = a_1 + a_2$, $S_3 = a_1 + a_2 + a_3$, ... $S_n = a_1 + a_2 + \dots + a_n$, ... _____ to a number S as n gets very large.

When an infinite series does not converge to a sum, the series _____. An infinite geometric series with $|r| \geq 1$ diverges.

Example 3: Analyzing Infinite Geometric Series

Does the series converge or diverge? If it converges, what is the sum?

a.) $-5 - \frac{5}{2} - \frac{5}{4} - \frac{5}{8} - \frac{5}{16} - \dots$

b.) $\frac{1}{4} - \frac{3}{8} + \frac{9}{16} - \frac{27}{32} + \dots$

c.) $\sum_{n=0}^{\infty} (0.8)^n$

